TTP99-11
February 1999

The second order QCD contribution to the semileptonic $b \to u$ decay rate

Timo van Ritbergen

Institut für Theoretische Teilchenphysik, Universität Karlsruhe
D-76128 Karlsruhe, GERMANY

Abstract

The order α_s^2 contribution to the inclusive semileptonic decay width of a b quark $\Gamma(b \to X_u e \overline{\nu}_e)$ is calculated analytically for zero mass u quarks

PACS: 12.15.Hh, 12.38.Bx, 13.20.-v, 13.30.Ce

Measurements of the semileptonic B meson decay rates at B factories [1] provide precise means to extract the Cabibbo-Kobayashi-Maskawa matrix elements $|V_{ub}|$ and $|V_{cb}|$ from experiment. The accurate determination of these parameters is a requisite for many tests of CP violation within the three-generation Standard Model.

The Heavy Quark Expansion provides a framework for the systematic calculation of contributions to the inclusive B meson decay rate [2–6]. To leading order in the heavy quark mass the inclusive B meson decay rate is equal to the decay rate of a on-shell b quark treated within renormalization group improved perturbative QCD. The non-perturbative corrections are suppressed by at least two powers of the heavy quark mass and can be expressed as matrix elements of higher dimension operators in the heavy quark effective field theory.

Perturbative QCD contributions to b quark decay play an important role in accurate predictions for the inclusive B meson decay rate. The first order QCD corrections to the inclusive semileptonic $b \to u$ decay width $\Gamma(b \to X_u e \overline{\nu}_e)$ were derived from the calculation of the 1-loop QED corrections to the muon decay rate [7]. For $\Gamma(b \to X_c e \overline{\nu}_e)$ c quark mass corrections are important and the 1-loop QCD corrections were obtained in a closed analytical form in Ref. [8].

For dimensional reasons $\Gamma(b \to X_u e \overline{\nu}_e)$ is proportional to the b quark mass to the fifth power. The adopted definition for the b quark mass therefore strongly affects the coefficients of the higher order QCD corrections. If the width is expressed in terms of the $\overline{\mathrm{MS}}$ renormalized b quark mass one finds that the 1-loop QCD corrections to $\Gamma(b \to X_u e \overline{\nu}_e)$ are significant, one-third of the size of the tree level contribution.

The 2-loop QCD contribution to $\Gamma(b \to X_u e \overline{\nu}_e)$ and $\Gamma(b \to X_c e \overline{\nu}_e)$ proportional to the number of light quark flavours was calculated in Refs. [9, 10]. This result was used to obtain an estimate for the full 2-loop corrections for both semileptonic $b \to c$ and $b \to u$ decay [10]. Recently an accurate estimate of the full 2-loop correction to the inclusive semileptonic $b \to c$ decay rate was obtained [11] based on calculations at 3 distinct values for the invariant mass of the lepton pair [11–13].

In this letter the 2-loop QCD corrections to the inclusive semileptonic $b \to u$ decay rate are evaluated analytically. The starting point of the calculation is the Lagrangian

$$\mathcal{L} = \mathcal{L}_W + \mathcal{L}_{\mathrm{QCD}} \tag{1}$$

Here $\mathcal{L}_W$ is the effective weak interaction term

$$\mathcal{L}_W = -\frac{G_F}{\sqrt{2}} V_{ub} \big[\bar{\psi}_u \gamma_\lambda (1-\gamma_5) \psi_b\big] . \big[\bar{\psi}_{\nu_e} \gamma_\lambda (1-\gamma_5) \psi_e\big] \tag{2}$$

where ψ_u, ψ_b, ψ_e and ψ_{ν_e} are the wave functions for the u quark, b quark, electron and its neutrino respectively. $\mathcal{L}_{\mathrm{QCD}}$ is the standard QCD Lagrangian for the strong interactions. The calculation is performed in effective 5 flavour QCD since the t quark decouples for the process under consideration. The QCD correction to the semileptonic

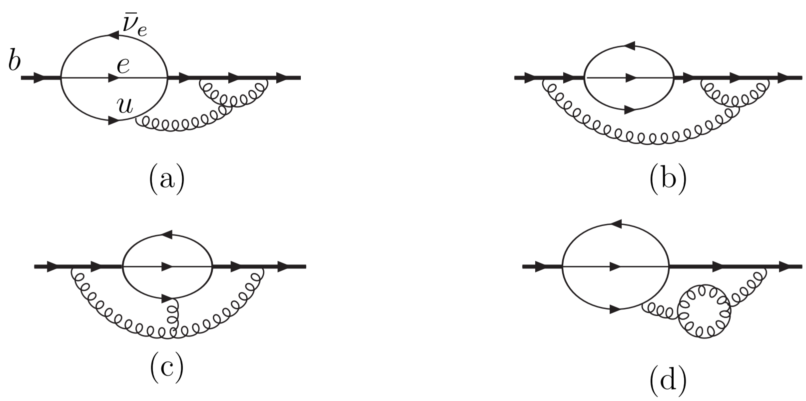

Figure 1: Examples of non-abelian diagrams whose cuts give contributions to $b \to ue\overline{\nu}_e$, $b \to ue\overline{\nu}_e g$, $b \to ue\overline{\nu}_e gg$.

decay rate are finite since they are separately finite for a vector quark current and the flavour non-singlet axial-vector quark current.

Throughout this article we use dimensional regularization [14, 15] and the standard modification of the minimal subtraction scheme [16], the $\overline{\text{MS}}$-scheme [17] for coupling constant renormalization. Calculations with massive external fermions are most easily performed in the on-shell renormalization scheme for fermion masses. Nevertheless, the final results in this letter will be expressed in terms of $\overline{\text{MS}}$ renormalized quark masses. For the treatment of the γ_5 matrix in dimensional regularization the technique described in Ref. [18] is used which is based on the original definition of γ_5 in Ref. [15]. It is interesting to note that in the limit of massless u quarks the contribution to the decay rate coming from the axial-vector part of the weak interaction Eq. (2) can be reduced to the vector part by using the effective anticommutation property of the γ_5 matrix.

For the order α_s^2 corrections to the inclusive width $\Gamma(b \to X_u e\overline{\nu}_e)$ one has to determine the sum of the decay widths $b \to ue\overline{\nu}_e$, $b \to ue\overline{\nu}_e g$, $b \to ue\overline{\nu}_e gg$ and $b \to ue\overline{\nu}_e q\overline{q}$, where g denotes a gluon and q denotes a quark, with up to two gluons or quarks in virtual corrections. According to the Kinoshita-Lee-Nauenberg [19] theorem, the inclusive decay rate is free from singularities as the u quark mass goes to zero. The calculation will be done in the approximation of massless u,d,s and c quarks. The c quark enters only in the partial rate $b \to ue\overline{\nu}_e c\overline{c}$ or as $c\overline{c}$ pairs in virtual corrections, and the effect of a non-zero c quark mass on the total decay rate is therefore limited. The situation is quite different for $\Gamma(b \to X_c e\overline{\nu}_e)$ where the c quark enters in all diagrams and c quark mass effects are very important.

Using the optical theorem one may express the inclusive decay rate as the imaginary part of 4-loop on-shell propagator type diagrams. Some of these diagrams are shown in Fig 1. The imaginary part of these 4-loop quark propagator diagrams can be evaluated

analytically using the methods employed in Ref. [20] where the 2-loop QED corrections to the muon decay rate were evaluated. As was noted before, the muon decay rate and $\Gamma(b \to X_u e \overline{\nu}_e)$ are closely related; the result for the 2-loop QED corrections to the muon decay width corresponds formally to keeping only the abelian part in the 2-loop QCD corrections to $\Gamma(b \to X_u e \overline{\nu}_e)$. The approach of Ref. [20] relies heavily on the use of the method of integration-by-parts [21] within dimensional regularization to express the imaginary part of the 4-loop diagrams in terms of a small set of 26 primitive integrals. Although there are diagrams for $\Gamma(b \to X_u e \overline{\nu}_e)$ that involve integration topologies that were absent in Ref. [20] it turns out that after the use of integration-by-parts identities for these topologies no new primitive integrals are needed.

All diagrams that contribute to $\Gamma(b \to X_u e \overline{\nu}_e)$ are calculated in a general covariant gauge for the gluon fields. The explicit cancellation of the gauge dependence in the sum of the diagrams gives an important check of the result. The result that is obtained in this way reads

$$\Gamma(b \to X_u e \overline{\nu}_e) = \frac{G_F^2 |V_{ub}|^2 M_b^5}{192\pi^3} \left[1 + b_1 \frac{\alpha_s^{(5)}(\mu)}{\pi} + b_2 \left(\frac{\alpha_s^{(5)}(\mu)}{\pi} \right)^2 + O(\alpha_s^3) \right] \tag{3}$$

$$\begin{aligned}
b_1 &= C_F \left(\tfrac{25}{8} - 3\zeta_2 \right) \\
&\approx -2.41307 \\
b_2 &= C_A C_F \left[\tfrac{154927}{10368} - \tfrac{53}{2}\zeta_2 \ln(2) + \tfrac{95}{27}\zeta_2 - \tfrac{383}{72}\zeta_3 + \tfrac{101}{16}\zeta_4 + \tfrac{275}{96} \ln\left(\tfrac{\mu^2}{M_b^2}\right) - \tfrac{11}{4}\zeta_2 \ln\left(\tfrac{\mu^2}{M_b^2}\right) \right] \\
&\quad + C_F^2 \left[\tfrac{11047}{2592} + 53\zeta_2 \ln(2) - \tfrac{1030}{27}\zeta_2 - \tfrac{223}{36}\zeta_3 + \tfrac{67}{8}\zeta_4 \right] \\
&\quad + C_F T_F n_f \left[-\tfrac{1009}{288} + \tfrac{77}{36}\zeta_2 + \tfrac{8}{3}\zeta_3 - \tfrac{25}{24} \ln\left(\tfrac{\mu^2}{M_b^2}\right) + \zeta_2 \ln\left(\tfrac{\mu^2}{M_b^2}\right) \right] \\
&\quad + C_F T_F \left(\tfrac{6335}{192} - \tfrac{9}{2}\zeta_2 - 24\zeta_3 \right) \\
&\approx -21.29553 - 4.625050 \ln\left(\tfrac{\mu^2}{M_b^2}\right)
\end{aligned} \tag{4}$$

where $\alpha_s^{(5)}(\mu)$ is the coupling constant in effective 5 flavour QCD, μ is the renormalization scale, $n_f = 5$ is the total number of quark flavours and M_b is the b quark pole mass in the two loop order. $C_F = 4/3$ and $C_A = 3$ are the Casimir operators of the fundamental and adjoint representation of the colour group SU(3), $T_F = 1/2$ is the trace normalization of the fundamental representation. ζ_k denotes the Riemann zeta function, $\zeta_2 = \pi^2/6$, $\zeta_3 = 1.2020569\cdots$, $\zeta_4 = \pi^4/90$.

We note that the coefficient of $C_F T_F n_f$ in Eq.(4) agrees with the value 3.22 obtained in Ref. [9] and the terms proportional to C_F^2 and $C_F T_F$ agree with the result for the QED contributions to the muon decay width of Ref. [20]. The order α_s^2 coefficient in Eq. (4) is sizable but this coefficient is somewhat smaller than the estimate -28.7 obtained in Ref. [10].

It has been shown [22,23] that the perturbative coefficients for $\Gamma(b \to X_u e \overline{\nu}_e)$ receive contributions that grow rapidly at higher orders corresponding to a singularity in the Borel plane at $\pi/(2\beta_0)$ (with β_0 given below Eq. (15)) when the decay width is expressed in terms of the b quark pole mass. This particular growth of the perturbative coefficients is related to the infrared sensitivity of the pole mass definition and is absent when the inclusive decay width is expressed in terms of the $\overline{\rm MS}$ renormalized b quark mass $m_b(\mu)$. We will therefore express the decay width in terms of $m_b(\mu)$ using the known relation between the pole quark mass and the $\overline{\rm MS}$ renormalized quark mass [24, 25].

$$M_b = m_b(\mu)\left[1 + c_1 \frac{\alpha_s^{(5)}(\mu)}{\pi} + c_2\left(\frac{\alpha_s^{(5)}(\mu)}{\pi}\right)^2 + O(\alpha_s^3)\right] \tag{5}$$

$$\begin{aligned} c_1 &= C_F\left(1 + \tfrac{3}{4}\ln\left(\tfrac{\mu^2}{M_b^2}\right)\right) \\ c_2 &= C_A C_F\left[\tfrac{1111}{384} + \tfrac{3}{2}\zeta_2\ln(2) - \tfrac{1}{2}\zeta_2 - \tfrac{3}{8}\zeta_3 + \tfrac{185}{96}\ln\left(\tfrac{\mu^2}{M_b^2}\right) + \tfrac{11}{32}\ln^2\left(\tfrac{\mu^2}{M_b^2}\right)\right] \\ &\quad + C_F^2\left[\tfrac{121}{128} - 3\zeta_2\ln(2) + \tfrac{15}{8}\zeta_2 + \tfrac{3}{4}\zeta_3 + \tfrac{27}{32}\ln\left(\tfrac{\mu^2}{M_b^2}\right) + \tfrac{9}{32}\ln^2\left(\tfrac{\mu^2}{M_b^2}\right)\right] \\ &\quad + C_F T_F n_f\left[-\tfrac{71}{96} - \tfrac{1}{2}\zeta_2 - \tfrac{13}{24}\ln\left(\tfrac{\mu^2}{M_b^2}\right) - \tfrac{1}{8}\ln^2\left(\tfrac{\mu^2}{M_b^2}\right)\right] + C_F T_F\left(-\tfrac{3}{4} + \tfrac{3}{2}\zeta_2\right) \end{aligned} \tag{6}$$

Applying this relation to eliminate M_b from Eq.(3) and from the logarithms appearing in Eqs. (4,6)

$$\ln\left(\frac{\mu^2}{M_b^2}\right) = \ln\left(\frac{\mu^2}{m_b^2(\mu)}\right) - 2C_F\left[1 + \frac{3}{4}\ln\left(\frac{\mu^2}{m_b^2(\mu)}\right)\right]\frac{\alpha_s^{(5)}(\mu)}{\pi} + O(\alpha_s^2) \tag{7}$$

one obtains the result for the decay width in terms of the $\overline{\rm MS}$ renormalized b quark mass

$$\Gamma(b \to X_u e \overline{\nu}_e) = \frac{G_F^2|V_{ub}|^2}{192\pi^3}\left(m_b^{(5)}(\mu)\right)^5\left[1 + h_1\frac{\alpha_s^{(5)}(\mu)}{\pi} + h_2\left(\frac{\alpha_s^{(5)}(\mu)}{\pi}\right)^2 + O(\alpha_s^3)\right] \tag{8}$$

$$
\begin{aligned}
h_1 &= C_F\left[\tfrac{65}{8} - 3\zeta_2 + \tfrac{15}{4}\ln\left(\tfrac{\mu^2}{m_b^2(\mu)}\right)\right] \\
&\approx 4.25360 + 5\ln\left(\tfrac{\mu^2}{m_b^2(\mu)}\right) \\
h_2 &= C_A C_F\left[\tfrac{19057}{648} - 19\zeta_2\ln(2) + \tfrac{55}{54}\zeta_2 - \tfrac{259}{36}\zeta_3 + \tfrac{101}{16}\zeta_4 + \tfrac{25}{2}\ln\left(\tfrac{\mu^2}{m_b^2(\mu)}\right)\right. \\
&\qquad \left. -\tfrac{11}{4}\zeta_2\ln\left(\tfrac{\mu^2}{m_b^2(\mu)}\right) + \tfrac{55}{32}\ln^2\left(\tfrac{\mu^2}{m_b^2(\mu)}\right)\right] \\
&\quad + C_F^2\left[\tfrac{281113}{10368} + 38\zeta_2\ln(2) - \tfrac{9455}{216}\zeta_2 - \tfrac{22}{9}\zeta_3 + \tfrac{67}{8}\zeta_4 + \tfrac{405}{16}\ln\left(\tfrac{\mu^2}{m_b^2(\mu)}\right)\right. \\
&\qquad \left. -\tfrac{45}{4}\zeta_2\ln\left(\tfrac{\mu^2}{m_b^2(\mu)}\right) + \tfrac{225}{32}\ln^2\left(\tfrac{\mu^2}{m_b^2(\mu)}\right)\right] \\
&\quad + C_F T_F n_f\left[-\tfrac{1037}{144} - \tfrac{13}{36}\zeta_2 + \tfrac{8}{3}\zeta_3 - \tfrac{15}{4}\ln\left(\tfrac{\mu^2}{m_b^2(\mu)}\right) + \zeta_2\ln\left(\tfrac{\mu^2}{m_b^2(\mu)}\right) - \tfrac{5}{8}\ln^2\left(\tfrac{\mu^2}{m_b^2(\mu)}\right)\right] \\
&\quad + C_F T_F\left(\tfrac{5615}{192} + 3\zeta_2 - 24\zeta_3\right) \\
&\approx 26.78476 + 36.99016\ln\left(\tfrac{\mu^2}{m_b^2(\mu)}\right) + 17.29167\ln^2\left(\tfrac{\mu^2}{m_b^2(\mu)}\right) \qquad (9)
\end{aligned}
$$

where $m_b(\mu) = m_b^{(5)}(\mu)$ is the $\overline{\mathrm{MS}}$ renormalized b quark mass in effective 5 flavour QCD. The decay width may also be expressed in terms of parameters of 4 flavour QCD by using the known decoupling relations for the coupling constant and the quark masses [26–28]

$$
\begin{aligned}
\frac{\alpha_s^{(5)}(\mu)}{\pi} &= \frac{\alpha_s^{(4)}(\mu)}{\pi} + \left(\frac{\alpha_s^{(4)}(\mu)}{\pi}\right)^2 \frac{T_F}{3}\ln\left(\frac{\mu^2}{m_b^2(\mu)}\right) + O(\alpha_s^3) \qquad (10) \\
m_q^{(5)}(\mu) &= m_q^{(4)}(\mu)\left[1 + \left(\frac{\alpha_s^{(4)}(\mu)}{\pi}\right)^2 T_F C_F\left(-\frac{1}{8}\ln^2\left(\frac{\mu^2}{m_b^2(\mu)}\right)\right.\right. \\
&\qquad \left.\left. + \frac{5}{24}\ln\left(\frac{\mu^2}{m_b^2(\mu)}\right) - \frac{89}{288}\right) + O(\alpha_s^3)\right] \qquad (11)
\end{aligned}
$$

to obtain

$$
\Gamma(b \to X_u e\overline{\nu}_e) = \frac{G_F^2|V_{ub}|^2}{192\pi^3}\left(m_b^{(4)}(\mu)\right)^5\left[1 + f_1\frac{\alpha_s^{(4)}(\mu)}{\pi} + f_2\left(\frac{\alpha_s^{(4)}(\mu)}{\pi}\right)^2 + O(\alpha_s^3)\right] \qquad (12)
$$

$$
\begin{aligned}
f_1 &\approx 4.25360 + 5\ln\left(\tfrac{\mu^2}{m_b^2(\mu)}\right) \\
f_2 &\approx 25.75467 + 38.39353\ln\left(\tfrac{\mu^2}{m_b^2(\mu)}\right) + 17.70833\ln^2\left(\tfrac{\mu^2}{m_b^2(\mu)}\right). \qquad (13)
\end{aligned}
$$

Here $m_b(\mu) = m_b^{(4)}(\mu)$ and only the numerical expressions are given.

Each of the obtained expressions for the decay width Eqs. (3),(8) and (12) satisfies formal renormalization group invariance that is required of a physical quantity which in the α_s^2 approximation reads

$$\frac{d}{d\ln(\mu^2)}\Gamma(\mu^2,\alpha_s(\mu),m_b(\mu)) = O(\alpha_s^3(\mu)) \tag{14}$$

The μ dependence in the renormalized mass and coupling constants is given by the renormalization group equations

$$\frac{da}{d\ln\mu^2} = \beta(a) = -\beta_0 a^2 - \beta_1 a^3 - \beta_2 a^4 - \beta_3 a^5 + O(a^6) \tag{15}$$

$$\frac{d\ln m_q}{d\ln\mu^2} = \gamma_m(a) = -\gamma_0 a - \gamma_1 a^2 - \gamma_2 a^3 - \gamma_3 a^4 + O(a^5) \tag{16}$$

where $a = \alpha_s/\pi$, $\beta_0 = \frac{11}{12}C_A - \frac{1}{3}T_F n_f$, $\gamma_0 = \frac{3}{4}C_F$, $\gamma_1 = C_F(\frac{3}{32}C_F + \frac{97}{96}C_A - \frac{5}{24}T_F n_f)$ and n_f is the number of quark flavours in effective QCD. The higher order coefficients up to β_3 and γ_3 are given in [29].

As is indicated in Eq. (14) the two loop QCD expressions for $\Gamma(b\to X_u e\overline{\nu}_e)$ have a remaining renormalization scale dependence that is of order α_s^3. The renormalization scale dependence of the expression for the decay width Eq. (8) in the different orders of perturbative QCD is illustrated in Fig 2. In Fig. 2 the leading order curve (LO) corresponds to the tree level expression for the decay width and to solving the renormalization group equation for $\alpha_s(\mu)$ and $m_b(\mu)$ in the leading order, the next-to-leading order curve (NLO) corresponds to keeping one higher order in α_s both in the expression for the decay width and the renormalization group equations, etc. The renormalization group equation for $\alpha_s(\mu)$ is solved numerically starting from $\alpha_s^{(5)}(m_Z) = 0.118$. Similarly the scale evolution of $m_b^{(5)}(\mu)$ is done numerically starting from $m_b^{(5)}(m_b) = 4.3$ GeV. For comparison we have included a NNNLO curve that corresponds to taking the presently unknown constant of order α_s^3 in Eq. (8) to be $5^3 = 125$, assuming here an approximate power-like growth of the lower order coefficients, see e.g. Ref. [30]. The logarithmic terms $\alpha_s^3\ln^i(\mu^2/m_b^2(\mu))$, $i = 1,2,3$ can be expressed in terms of the known lower order coefficients by requiring formal μ-independence of the decay rate in the α_s^3 order.

One may see from Fig. 2 that in the NNLO expression Eq. (8) a moderate renormalization scale dependence remains near $\mu = m_b(m_b)$. Variation of μ in the relatively wide interval between $\mu = 1/2\, m_b(m_b)$ and $\mu = 2\, m_b(m_b)$ changes the decay width in the NNL order by about 15%. If variation with respect to μ in this interval is used to estimate the contributions to $\Gamma(b\to X_u e\overline{\nu}_e)$ beyond the α_s^2 order one finds that these unknown higher order contributions should not hinder the extraction of $|V_{ub}|$ down to an accuracy of 8%. Furthermore we note that the difference between the LO,NLO and NNLO curves is minimal for $\mu \approx 2.5$ GeV, i.e. the apparent convergence of the perturbation series is optimal for this choice of the renormalization scale (which is within the above mentioned interval). This fact is not surprising as the size of the coefficients

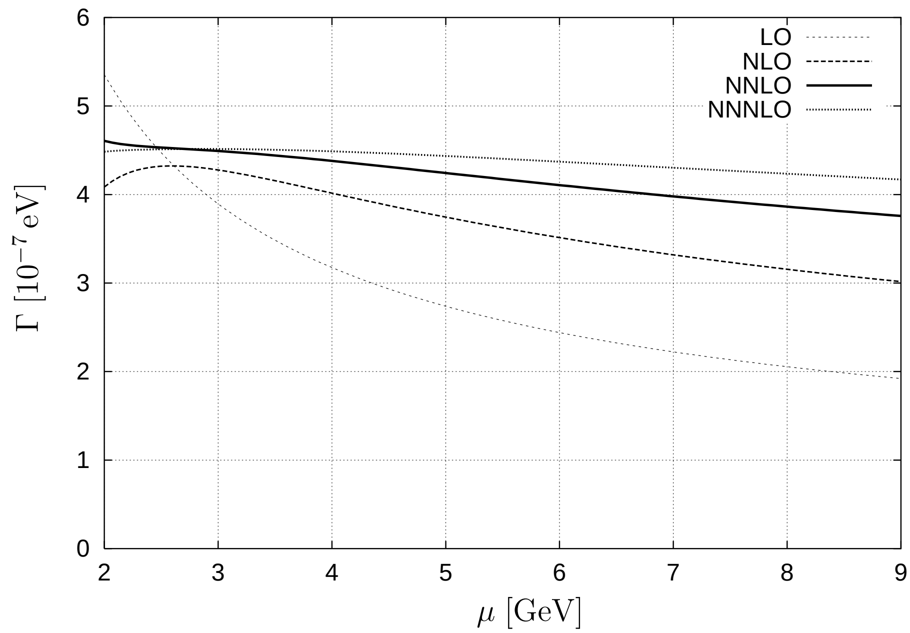

Figure 2: Renormalization scale dependence of $\Gamma(b \to X_u e \overline{\nu}_e)$ near $\mu = m_b(m_b)$ in different orders of perturbative QCD for $\alpha_s^{(5)}(m_Z) = 0.118$, $m_b^{(5)}(m_b) = 4.3\,\mathrm{GeV}$ and $|V_{ub}| = 0.003$.

h_1 and h_2 in Eq. (8) is greatly reduced for this value of μ. It is also in agreement with the observation [30] that a more natural renormalization scale for the $\overline{\mathrm{MS}}$ renormalized b quark mass in inclusive semileptonic b decay is somewhat below $m_b(m_b)$ as the scale is set by the characteristic energy that is released into the hadronic final state. Adopting $\mu \approx 2.5\,\mathrm{GeV}$ as a choice for the renormalization scale we conclude that the α_s^2 order approximation to $\Gamma(b \to X_u e \overline{\nu}_e)$ that is obtained in this letter provides a good theoretical foundation for the extraction of $|V_{ub}|$ from inclusive semileptonic B meson decays.

Useful discussions with K. Chetyrkin, A. Czarnecki, Th. Mannel, K. Melnikov and Y.-P. Yao are gratefully acknowledged. This work was supported in part by BMBF under contract No. 057KA92P and DFG Forschergruppe under contract KU 502/8-1.

References

[1] For an overview of physics at B factories see e.g. "The BABAR Physics Book", SLAC Report 504 (Stanford, 1998).

[2] M. Voloshin and M. Shifman, *Sov. J. Nucl. Phys.* **41** (1985) 120.

[3] J. Chay, H. Georgi and B. Grinstein, *Phys. Lett.* **B247** (1990) 399.

[4] I. Bigi, N.G. Uraltsev and A.I. Vainshtein, *Phys. Lett.* **B293** (1992) 430;
I. Bigi, M. Shifman, N.G. Uraltsev and A.I. Vainshtein, *Phys. Rev. Lett.* **71** (1993) 496.

[5] A.F. Falk and M. Neubert, *Phys. Rev.* **D47** (1993) 2965; ibid. **D47** (1993) 2982.

[6] A.V. Manohar and M.B. Wise, *Phys. Rev.* **D49** (1994) 1310;
B. Blok et al., *Phys. Rev.* **D49** (1994) 3356 [(E) ibid. **D50** (1994) 3572];
T. Mannel, *Nucl. Phys.* **B413** (1994) 396.

[7] T. Kinoshita and A. Sirlin, *Phys. Rev.* **113** (1959) 1652;
S. M. Berman, *Phys. Rev.* **112** (1958) 267.

[8] Y. Nir, *Phys. Lett.* **B221** (1989) 184;
M. Jezabek and J.H. Kühn, *Nucl. Phys.* **B314** (1989) 1.

[9] M. Luke, M.J. Savage and M.B. Wise, *Phys. Lett.* **B343** (1995) 329.

[10] M. Luke, M.J. Savage and M.B. Wise, *Phys. Lett.* **B345** (1995) 301.

[11] A. Czarnecki and K. Melnikov, preprint BNL-HET-98-12 (1998), hep-ph/9804215.

[12] A. Czarnecki and K. Melnikov, *Nucl. Phys.* **B505** (1997) 65;
J. Franzkowski and J.B. Tausk, *Eur. Phys. J.* **C5** (1998) 517.

[13] A. Czarnecki and K. Melnikov, *Phys. Rev. Lett.* **78** (1997) 3630.

[14] C.G. Bollini and J.J. Giambiagi, *Phys. Lett.* **40B** (1972) 566.

[15] G. 't Hooft and M. Veltman, *Nucl. Phys.* **B44** (1972) 189.

[16] G. 't Hooft, *Nucl. Phys.* **B61** (1973) 455.

[17] W.A. Bardeen, A.J. Buras, D.W. Duke and T. Muta, *Phys. Rev.* **D18** (1978) 3998.

[18] S.A. Larin, *Phys. Lett.* **B303** (1993) 113.

[19] T. Kinoshita, *J. Math. Phys.* **3** (1962) 650;
T.D. Lee and M. Nauenberg, *Phys. Rev.* **133** (1964) 1549.

[20] T. van Ritbergen and R.G. Stuart, *Phys. Rev. Lett.* **82** (1999) 488.

[21] K.G. Chetyrkin and F.V. Tkachov, *Nucl. Phys.* **B192** (1981) 159.

[22] I. Bigi, M. Shifman, N. Uraltsev and A. Vainshtein, *Phys. Rev.* **D50** (1994) 2234.

[23] M. Beneke, V.M. Braun and V.I. Zakharov, *Phys. Rev. Lett.* **73** (1994) 3058; P. Ball, M. Beneke and V.M. Braun, *Phys. Rev.* **D52** (1995) 3929.

[24] N. Gray, D.J. Broadhurst, W. Grafe and K. Schilcher, *Z. Phys.* **C48** (1990) 673; D.J. Broadhurst, N. Gray and K. Schilcher, *Z. Phys.* **C52** (1991) 111.

[25] J. Fleischer, F. Jegerlehner, O.V. Tarasov and O.L. Veretin, *Nucl. Phys.* **B539** (1999) 671.

[26] B. Ovrut and H. Schnitzer, *Nucl.Phys.* **B189** (1981) 509.

[27] W. Bernreuther and W. Wetzel, *Nucl. Phys.* **B197** (1982) 228; W. Bernreuther, *Annals of Physics*, **151** (1983) 127.

[28] S.A. Larin, T. van Ritbergen and J.A.M. Vermaseren, *Nucl. Phys.* **B438** (1995) 278; K.G. Chetyrkin, B.A. Kniehl and M. Steinhauser, *Phys. Rev. Lett.* **79** (1997) 2184.

[29] S.A. Larin, T. van Ritbergen and J.A.M. Vermaseren, *Phys. Lett.* **B400** (1997) 379; *Phys. Lett.* **B405** (1997) 327; K.G. Chetykin, *Phys. Lett.* **B404** (1997) 161.

[30] I. Bigi, M. Shifman, N. Uraltsev and A. Vainshtein, *Phys. Rev.* **D56** (1997) 4017.